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Transient Phenomena in the Marx Multiplying Circuit after Firing the First Coupling Spark Gap

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1. Introduction

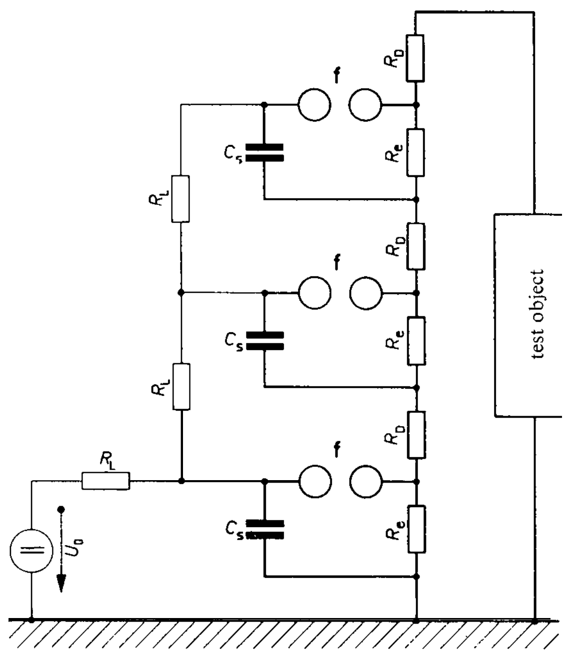
The Marx multiplying circuit, invented in 1923. has gained wide acceptance in scientific laboratories as well as in industrial test areas as the most practical means for producing high impulse voltages. In a Marx multiplying circuit, n capacitors connected in parallel are charged to a voltage U_0 by a DC voltage source, through high ohmic resistors (see Fig. 1). These capacitors are then switched in series by the firing of coupling spark gaps (CSG). Because of the series connection, an impulse voltage with a crest value of approximately $n\ U_0$ is obtained. The gap lengths of the CSG are chosen in such a way that the length of the gap in the first stage is smaller than those in all the other stages. When the charging voltage is slowly raised, this first CSG breaks down as soon as the voltage reaches its static breakdown value. Experience shows that after the firing of the first CSG, the CSG in the other stages will fire also and thus establish the series connections of all the capacitors in the impulse generator. In this paper, we shall examine the transient phenomena which follow the firing of the first CSG and which cause the firing of the others.

2. Earlier Investigations

Until now, the firing of the CSG has been described in the literature only in a very general way. Very little experimental work has been performed. The author knows of only two papers which discuss the transient phenomena based upon experimental results: *Elmer* [1] [1]) has analyzed the self-oscillations of the circuit after the firing of all the CSG. *Edwards, Husbands,* and *Perry* [2] have observed that, after the firing of the CSG, the earth capacitance of the generator is charged and that the necessary charges flow through the interstage resistors. During the charging of the earth capacitance, overvoltages are produced on the second and successive CSG. The duration of these overvoltages depends upon the value of the capacitance and of interstage resistors. *Edwards, Husbands,* and *Perry* have successfully recorded the transient voltages produced after the firing of the first CSG: but the results of their tests still do not give a complete picture of the firing process. They have not been able to prove directly that the overvoltages are effectively causing the firing of the other CSG.

3. The measuring arrangement

Because of the few known test results available, it was necessary firstly to enlarge the basis of experiments by our own measurements. Although the author has collected a considerable amount of data, discussions in this paper are limited to the circuit arrangement, now widely used, pro-



Eig. 1
Schematic diagram of a 3-stage multiplying circuit according to Marx
Arrangement of resistors according to ***Edwards*** and *Scoles*: $R_L \gg R_e \gg R_D$
C_s = impulse capacitor; f = CSG; R_e = discharging resistor; R_L = charging resistor; R_D = damping resistor; U_o = charging voltage

[1]) See literature at the end of the paper.

posed by *Edwards* and *Scoles*. The circuit arrangement is represented in Fig. 1 and is characterized by the fact that the impulse capacitors discharge through discharging resistors R_e placed at the different stages and that the discharging current does not flow through the damping resistors R_D in each stage. The charging resistors R_L are of much higher value than the discharging resistors R_e. The tests described below were performed without the damping resistors, in order to limit the number of parameters to be investigated. Check measurements have shown that the influence of such resistors is negligible when R_e is at least ten times larger than **RD,** which is usually the case.

For the experimental study of transient phenomena, a small six-stage impulse generator was built with an overall height of only 85 cm, a total charging voltage of 72 kV, and an energy of approximately 17 Ws. The self-inductance of one generator stage is 5 μH, which is rather high, considering the stage dimensions. This choice was necessary in order to adapt the frequencies of the generator self-oscillations to the limited frequency range of the measuring apparatus. The study is limited first to the transient phenomena after the voltage collapse on the first CSG. It is useful to study these phenomena and temporarily leave aside the firing of the other CSG. If other CSG do not fire, the transient phenomena are not influenced by elements depending upon the voltage if we assume that all capacitors and resistors are linear. It is thus possible to operate with any charging voltage. For most of the tests described below, the charging voltage amounts to only a few volts. The voltage collapse at the first CSG is produced by a mercury-contact switch. The voltage drops to zero in 10^{-9}s with this switch; that is, almost as quickly as in the generator normal operation by spark firing. The advantages of performing the tests at such very low voltages are: absence of hazards caused by high voltage, and the fact that it is unnecessary to consider the dielectric strength of the components in the Marx circuit. However, the principal advantage of this method is the continuous operation rendered by the mercury-contact switch, which results in a stable picture on the oscilloscope screen.

Fig. 2 shows the circuit arrangement of all the tests at low voltage. The ground terminal of the multiplying circuit is connected to a flat, copper earth electrode, while the HV terminal goes to the test object. Near the generator, a narrow vertical earth electrode serves as a ground for the voltage dividers. The transient phenomena caused by the closing of the mercury-contact switch are recorded on the oscilloscope through two *RC* voltage dividers. The two coaxial cables of the low voltage part of the dividers connect to the inputs of the oscilloscope differential amplifier. With one of the cathode-ray tube beams, it is possible to record either of the input voltage variations or their difference. The oscilloscope is connected to the earth electrode of the impulse generator only by the screening of the two coaxial cables. As only the transient phenomena of the first stage are presently under consideration, only the impulse capacitor of the first stage is charged for all low-voltage tests. The charging resistors leading to the upper stages are removed. Check measurements have proved that this arrangement has no noticeable influence on the transient voltages to be measured.

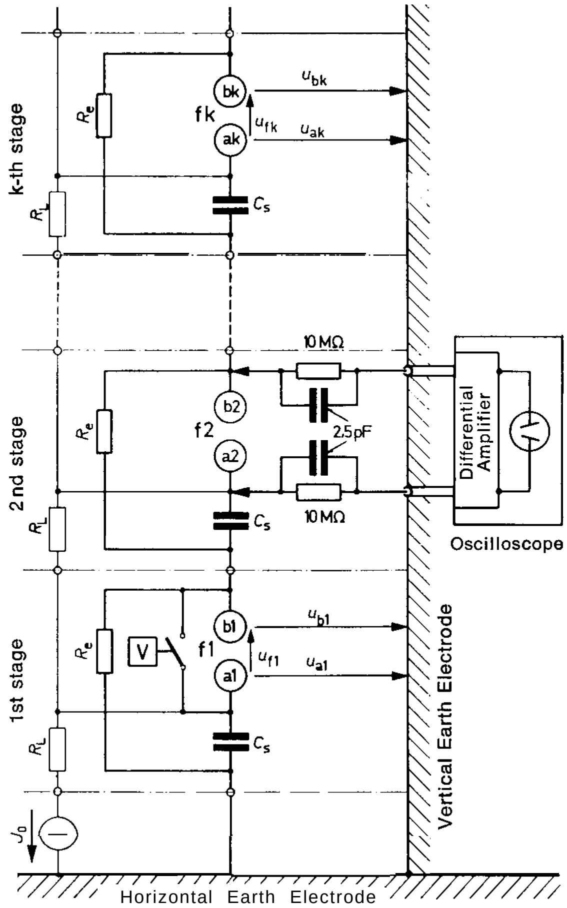


Fig. 2
Testing Circuit Arrangement daring voltage measurement at the second stage
Designations: see Fig. 1 and Item 5.

4. Accuracy of the measurements

Measurement errors arise in two ways: first, the accuracy of the measuring instrument is limited; second, the measuring equipment has an influence on the measured phenomena.

The restricted accuracy of the recording oscilloscope is due mainly to the response curve of the oscilloscope amplifiers, with the result that distorted impulse voltages are transmitted by the measuring system. Thus, for instance, the voltage collapse duration of the mercury-contact in the first CSG is approximately 10^{-9}s. The distortion of such a square-wave voltage pulse is taken generally as an indication of the transmission impulse ratio. In the case considered, the oscillogram of this phenomenon indicates a front duration of 2.10% (voltage $\ddot{u}_{f1}$, Fig. 3).

It is not possible to know whether measuring equipment with such a restricted impulse response ratio is adequate for the intended tests unless the range of frequencies of the phenomena to be measured is known. We have, therefore, conducted check measurements with another system with much larger band width (1000 MHz) and ascertained that the essential phenomena can be adequately measured with the equipment described.

The experiments mentioned below show that the transient phenomena resulting from the firing of the first CSG depend greatly upon the stray capacitances acting on the

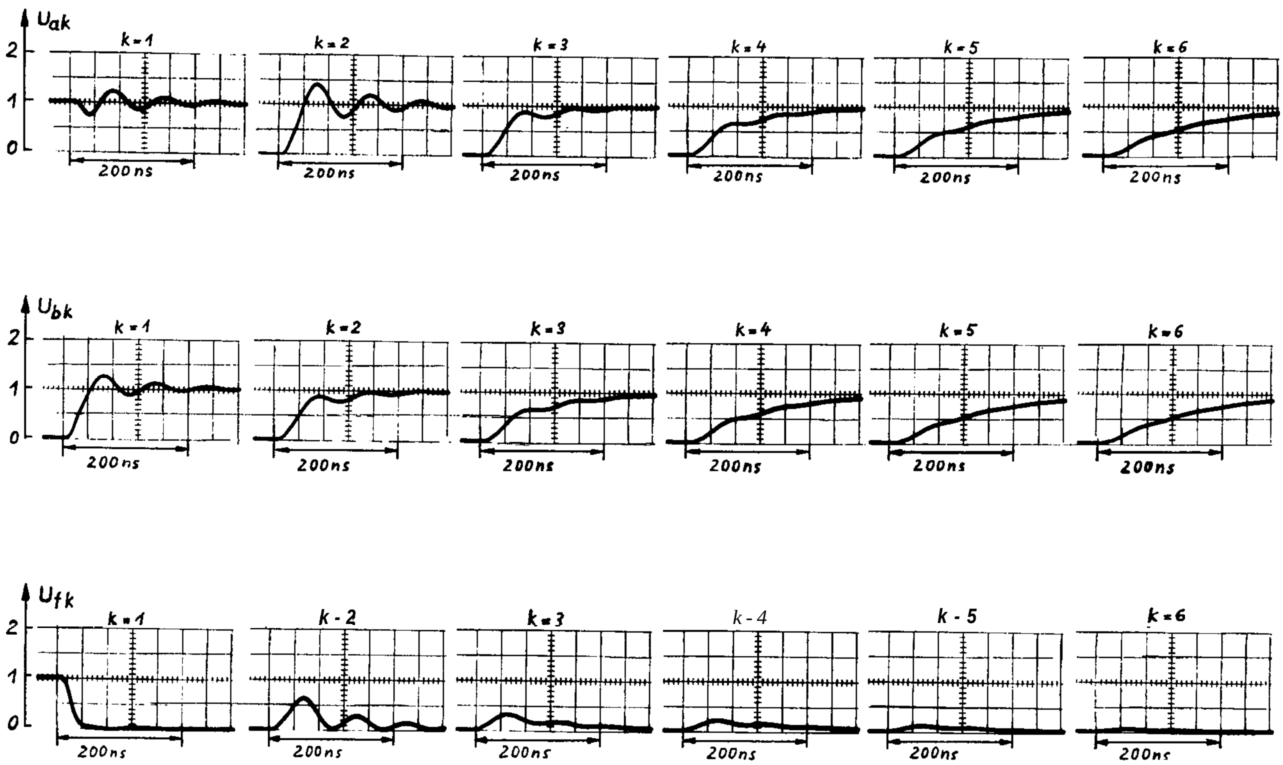


Fig. 3
CSG voltages to earth (u_{ak} and u_{bk}), CSG voltages (u_{fk}) of the 6-stage generator operating at no-load
$R_e = 1{,}75\ k\Omega$

impulse generator. Further, it is not possible to make measurements on the impulse generator without influencing these capacitances, specifically the earth capacitance. Therefore, the voltage dividers used for the experiments had very low capacitances; and moreover, the stray capacitances, not present with the normal operation of the generator, have been kept as low as possible.

5. Designations

To facilitate the analysis of the test result, a system of identification of the components in the Marx circuit was set up. The different generator stages bear successive numbers, starting with the stage connected to the earth electrode (Fig. 2). The spheres of the CSG having a DC voltage compared to earth are designed by ak when the generator is charged but has not yet fired ($k = 1, 2, \ldots$ =number of stages of the generator). The spheres earthed through discharging resistors during the charging process are designed by bk. For instance, a5 is the sphere in the fifth stage at charging potential while b5 is the sphere of the same stage but at earth potential until the firing moment. The voltage between a sphere and the earth electrode bears an index. Thus, for instance, u_{b4} is the voltage present between the earth electrode and the CSG of the fourth stage until the firing moment.

The CSG of the different stages bear the following designations: fl, f2, etc. The transient voltages appearing at these CSG during the firing process are designed by $\ddot{u}_{f1}$, $\ddot{u}_{f2}$, etc. The total voltage on the CSG, including the DC charging voltage, is designed as u_{fk}.

We thus have

$$u_{fk} = U_0 + \ddot{u}_{fk}$$

All the voltage scales in the oscillograms refer to the charging voltage U_0.

6. Measurement of Voltage Distribution in the Generator

Fig. 3 represents the voltage oscillograms in the generator at no load, after the voltage collapse on CSG fl. For this test, only the impulse capacitor of the first stage is charged from the DC voltage source. Therefore, only sphere al is at potential 1 compared with the earth electrode. The corresponding spheres of stages 2 to 6, a2 to a6, remain at earth potential up to the beginning of the transient phenomena. Voltage u_{a1} between earth electrode and sphere al has the initial value 1 and voltages u_{a2} to u_{a6} start from initial value 0. The first series of oscillograms in Fig. 3 shows the variation of voltages u_{a1} to u_{a6}. It can be remarked that voltage u_{a1} decreases slightly at the start of the transient phenomena, then returns to value 1 by damped oscillations. Voltage u_{a2} between earth electrode and sphere a2 of the second stage increases from zero, exceeds value 1 and then returns to this value by damped oscillations. Voltages u_{a3} to u_{a6} tend also from zero to value 1, but always more slowly when going to the upper stages.

The second series of oscillograms in Fig. 3 shows the voltages between earth electrode and spheres bl to b6. Just before the firing of CSG fl, these spheres remain at the earth electrode potential. After the voltage collapse on fl, voltage u_{b1} between earth electrode and sphere bl increases, exceeding value 1 by damped oscillations. Corresponding voltages at the upper stages, u_{b2} to u_{b6}, tend also from zero to value 1 but more slowly when going to the upper stages.

With the time bases used for these tests, the value of the voltage resulting after approximately 1 μs appears as the "final value" of the voltage variation. In the following, we shall use the expression "final value" in that sense, although for durations exceeding 1 μs all the voltages in the generator tend to zero.

The discharging process taking place with a half-wave duration of 50 μs is very slow compared with the transient phenomena (which occur immediately after the firing of the first CSG) treated in this paper.

The third series of oscillograms in Fig. 3 shows the transient phenomena appearing on the CSG. As only the impulse capacitor of the first stage is charged in this connection arrangement, only the transient voltages due to the firing of CSG f1 will appear on CSG f2 to f6 of the second to sixth stages. Normally, when all impulse capacitors are charged, these transient voltages are superimposed on the DC voltage on the CSG.

They should, therefore, be added to voltage 2 in normalized representation used here. Voltages $\ddot{u}_{f2}$ to $\ddot{u}_{f6}$ attain their crest value in less than approximately 40 ns and then decrease to zero in approximately 200 ns. Furthermore, a strong oscillation superimposes on the voltage of the second CSG with the result that the crest value of $\ddot{u}_{f2}$ attains 60 % of the stage charging voltage. The voltages on the other CSG are practically free from oscillations, and their crest values decrease when going to the upper stages.

Considering the first and second series of oscillograms in Fig. 3, it is further established that voltages u_{bk} correspond, with good approximation, to the voltages $u_{a(k+1)}$. This means that the overvoltages $\ddot{u}_{fk}$ are practically identical to the voltage drops through the corresponding discharging resistors:

$$\ddot{u}_{f(k+1)} = u_{bk} - u_{b(k+1)} \qquad (1)$$

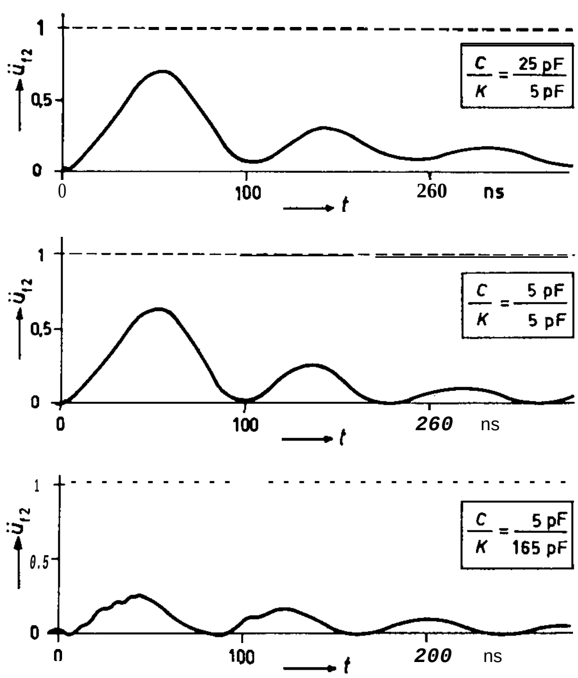


Fig. 4
Overvoltages ($\ddot{u}_{f2}$) at the second CSG, with different capacitance conditions to earth and longitudinal capacitances of the 6-stage generator operating at no-load
R_e = 1,75 kΩ; C = earth capacitance; K = longitudinal capacitance; t = time.

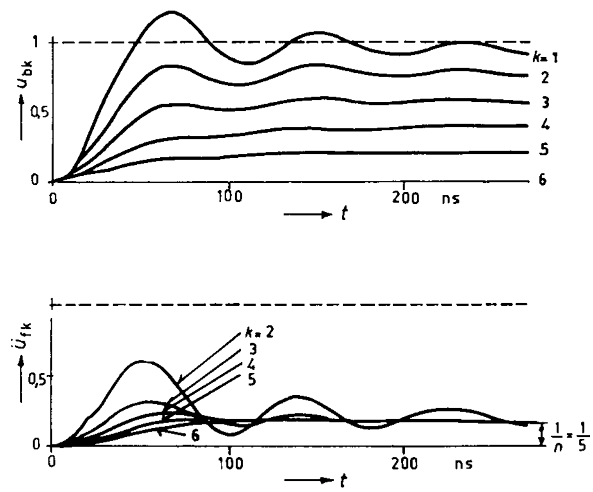


Fig. 5
Overvoltages ($\ddot{u}_{fk}$) and voltages to earth ($\ddot{u}_{bk}$) of the 6-stage short-circuited generator
R_e = 1,75 kΩ
Designations: see Fig. 1

In other words, the voltage distribution in the discharge resistor chain is directly proportional to the overvoltages in the CSG. The fact that the voltage distribution along the generator is irregular at first and only becomes uniform after some time lag, is due to the influence of generator capacitance with respect to earth. An artificial increase of this capacitance by means of an earthed screen placed in the close vicinity of the generator confirms this statement (Fig. 4a). Furthermore, as shown on Fig. 4c, an artificial increase of the stray capacitance between the generator stages causes a much more uniform voltage distribution along the generator and, therefore, less overvoltages at the CSG.

All the earlier measurements have been carried out on the generator under no-load conditions. The voltage conditions in the generator are very different when its output is short-circuited. Fig. 5 shows clearly that through voltages u_{bk} and u_{fk} the voltage leap, starting from the first stage at the moment of firing, becomes progressively zero along the charging resistors in the generator stages which have not yet fired. Due to this fact, the overvoltages acting upon the CSG do not decrease to zero during the interval of time considered in Fig. 5. They tend, in this case, to "final values" which each attain onefifth of the charging voltage in the first stage. In general, after firing the first CSG, in a generator of $(n+1)$ stages, the resulting values of the final normalized overvoltages will be $1/n$.

Particularly interesting voltage conditions occur in the case of a capacitive load for the generator (Fig. 6), which is, in fact, the most frequent case in the laboratory tests. The charging capacitance acts initially as a short circuit. In a three-stage, short-circuited generator, the final values of the overvoltages would be equal to half the charging voltage U_0. As shown on the oscillograms taken with a short sweep time (Fig. 6), the overvoltages appear also to tend at first towards these final values. Nevertheless, the output voltage u_{b3} does not remain constantly zero, as in the case of the short-circuited generator. On the contrary, because of the charging of the load capacitance through discharging resistors R_{e1} and R_{e2}, it increases with time constant

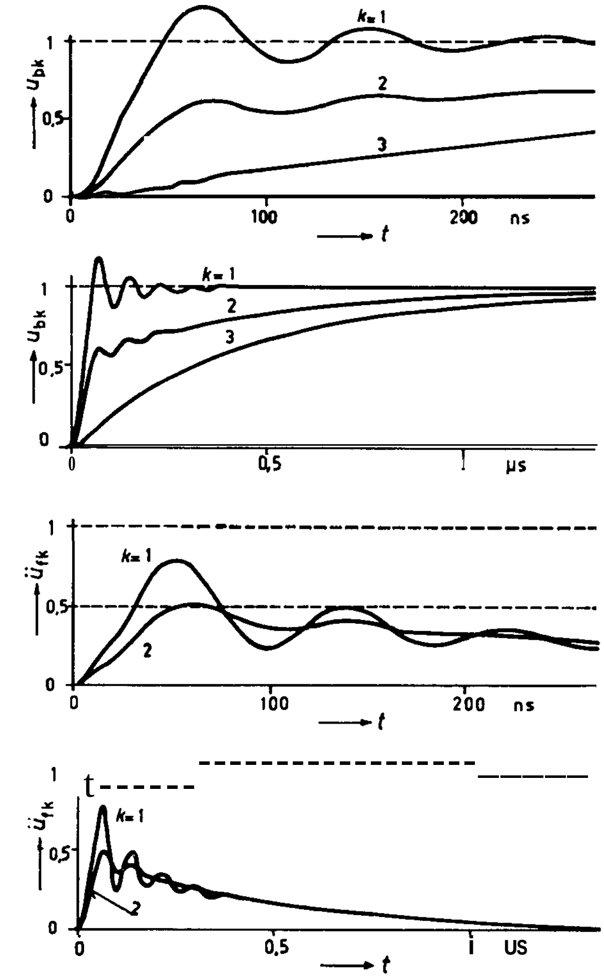


Fig. 6

Same as Fig. 5 but for a 3-stage capacitively loaded generator

Load capacitance C_B = 88 pF; discharging resistor R_e = 1,75 kΩ; t = time

Other designations: see Fig. 1

$$T_B = \frac{C_B\, C_s}{C_B + C_s}(R_{e1} + R_{e2})$$

The overvoltages at the second and third CSG decrease to zero with the same time constant. This phenomenon is only visible with a longer sweep time as shown on Fig. 6b.

7. The equivalent Diagram

The phenomena observed must be inserted in an equivalent diagram in order to proceed with calculation. The experiments have shown that the transient phenomena appearing after the firing of the first CSG depend essentially upon the discharging resistors, the generator stages, the stray capacitances to earth, the interstage stray capacitances, as well as the generator load. If we draw the stray capacitances in the diagram of a generator with $(n+1)$ stages, we obtain an iterative network as shown on Fig. 7. The first item of the chain is constituted by the first generator stage. The impulse capacitor C_{s1} charged at U_0 for this stage supplies the transient voltage after the firing of the first CSG fl. As this paper deals only with the phenomena following the firing of f1, no other impulse capacitor is discharged in the n other stages by the firing of a CSG. Therefore, for the equivalent diagram of the n stages which have not yet fired, there remain only the stray capacitances and the discharge resistor chain.

In Fig. 7, the longitudinal capacitances K and the earth capacitances C are uniformly distributed between all the generator stages in order to facilitate the calculation. Such a distribution represents only partially the real facts; and therefore, the calculation based upon this equivalent diagram can only be approximate. The mathematical resolution of this relatively simple diagram must show to what extent it is sufficient to describe quantitatively the results of the experiments.

If at first the self-inductance of the first stage is neglected (admittedly the impulse capacitance C_s of this stage is greater by many powers of ten than the stray capacitances C and K), the voltage u_{b1} at the beginning of the chain, representing the non-fired portion of the generator, has practically the form of a square wave impulse. The reaction of the chain to such an impulse can be easily calculated, basing our calculation upon the essays of *Wagner* [3]. This calculation concerns the short-circuited generator.

Because, in this case, the voltage conditions immediately following the firing of the first CSG are very analogous to those of the capacitively loaded generator, this calculation has a practical importance. Wagner gives the following equation for the voltage distribution along a chain of n links short-circuited at one end:

$$u_{bk}(p) = u_{b1}(p) \cdot \frac{\sinh\,[(n-k+1)g]}{\sinh n g} \qquad k = 1, 2, 3, \dots (n+1) \tag{2}$$

p is the complex variable in the range of the transformation according to Carson [3] used in the following:

$$C\{f(t)\} = p \int_0^\infty e^{-pt} \cdot f(t)\,.\,dt$$

The transition value g is related to the coupling elements of a link of the chain by the equation:

$$\cosh g = 1 + \frac{p R_e C}{2(1 + p R_e K)} \tag{3}$$

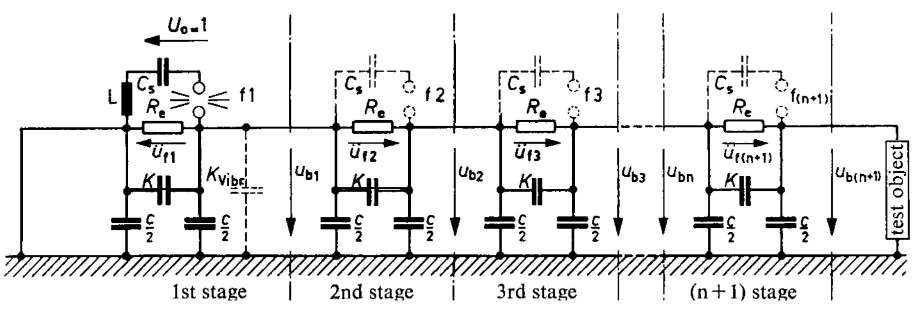


Fig. 7

Equivalent diagram of the Marx multiplying circuit for the calculation of the transient phenomena after the firing of the first CSG.

C_s = impulse capacitor; K = stray interstage capacitance; C = stray earth capacitance; $K_{vibr.}$ = stray capacitance of the mercury-contact switch; R_e = discharging resistance; U_o = charging voltage; $f_1, f_2 \dots f(n+1)$ = CSG; L = inductance of the first stage; $\ddot{u}_f$ = overvoltages on the CSG; u_b = voltages to earth of the chain of discharging resistors.

According to the development law of Heaviside, the following expression is derived from equations (1), (2) and (3) after some transformation:

$$\ddot{u}^{r}_{fk}(t) = \frac{1}{n} + \sum_{\mu=1}^{n-1} A_\mu \cdot e^{\frac{-t}{T_\mu}} \tag{5}$$

with

$$A_\mu = \frac{2\cos\left(\frac{2k-3}{2n}\mu\pi\right)\cos\left(\frac{\mu\pi}{2n}\right)}{n\left[1+\frac{2K}{C}\left(1-\cos\frac{\mu\pi}{n}\right)\right]} \tag{5a}$$

$$T_\mu = Re\left[K + \frac{C}{2\left(1-\cos\frac{\mu\pi}{n}\right)}\right] \tag{5b}$$

I’ indicates that these voltages are produced by a square wave impulse at the input of the chain. Equation (5) describes the overvoltage at the k-th CSG in a short-circuited generator with $(n+1)$ stages, after the firing of the first CSG, if it is assumed that the first stage has no self-inductance.

With the aid of a limit value, according to the Carson theory, an initial repartition of voltages can be drawn from equation (2). The limit value is

$$\lim_{p\to\infty} C\left\{f\ (t)\right\} = \lim_{t\to+0} f(r)$$

By applying this law to equation (2) and assuming that $\frac{C}{K}$ is less than 1, the following relation is obtained:

$$\lim_{t\to+0} u^{r}_{bk}(t) = \frac{\sinh\left[(n-k+1)\sqrt{\frac{C}{K}}\right]}{\sinh\left(n\sqrt{\frac{C}{K}}\right)} \tag{6}$$

This relation is identical to the well known approximate formula concerning the voltage distribution in a chain of insulators or for the initial repartition in a transformer winding or an impulse voltage divider.

Equation (5), obtained in very simplified conditions nevertheless describes some important features of transient phenomena (Fig. 8): According to the stray capacitances ratio ***C/K,*** the overvoltages attain their initial values at time $t = 0$. Starting from this capacitive initial voltage distribution, an exponential transition is obtained at the final values $1/n$. The transition duration depends, as shown by equation (5b), upon the value of the discharging resistors R_e and stray capacitances C and ***K.*** For the design of an impulse voltage generator, its impulse capacitance C_s and the required value of impulse voltage are always the determining factors. The stray capacitances automatically result from these factors, and very few possibilities are left for influencing them in order to obtain high overvoltages at the CSG. This is not the case with the discharging resistors R_e. Their function consists of discharging the impulse capacitors C_s within a prescribed period of time, for instance, for a time to half value of 50 μs, and can be partly assumed by a resistor connected in parallel with the whole generator.

It is thus possible to choose a high ohmic value for the discharging resistors in order to obtain high values of overvoltages, at the CSG with the longest possible duration, as long as the charging duration and voltage efficiency permit. It is, nevertheless, useless to choose values higher than charging resistors R_L.

The oscillograms reproduced in Fig. 8 show that the discharging resistors R_e not only influence the exponential transition between initial and final repartition but also play an important part in the damping of the oscillating portion of the overvoltage. The following calculation, taking into consideration the inductance of the first stage, gives more information on this subject. Impulse capacitor C_{s1} charged at voltage $U_0 = 1$ is replaced by a battery at voltage 1, as in the case of the evaluation of the chain behaviour with a square wave impulse. The capacitor self-inductance and a stage structure capacitance are lumped into inductance ***L.*** Capacitance K_1 is composed of longitudinal capacitance ***K*** and earth capacitance C of the first stage. If necessary, K_1 comprises also the earth capacitance of the mercury-contact switch.

If we consider that the input Impedance of a short-circuited chain with n links is Z . tgh n g, we obtain for tensibn u_{b1} (y) in the equation representation:

$$\frac{U_0(p)}{p\,L} - u_{b1}\left(\frac{1}{p\,L} + \frac{1}{R_e} + p\,K_1\right) = u_{b1}\frac{1}{Z(p)}\,\mathrm{ctgh}\,[n\,.\,g\,(p)] \tag{7}$$

The chain characteristic impedance ***Z(p)*** is calculated according to the structure of one of its links:

$$Z(p) = \frac{1}{p\sqrt{CK + \frac{C^2}{4} + \frac{C}{p\,R_e}}} \tag{8}$$

For this paper. the first stage of the overvoltage, i.e., its front and crest value. is the most pertinent. As shown below by the comparison between the calculation and experimental

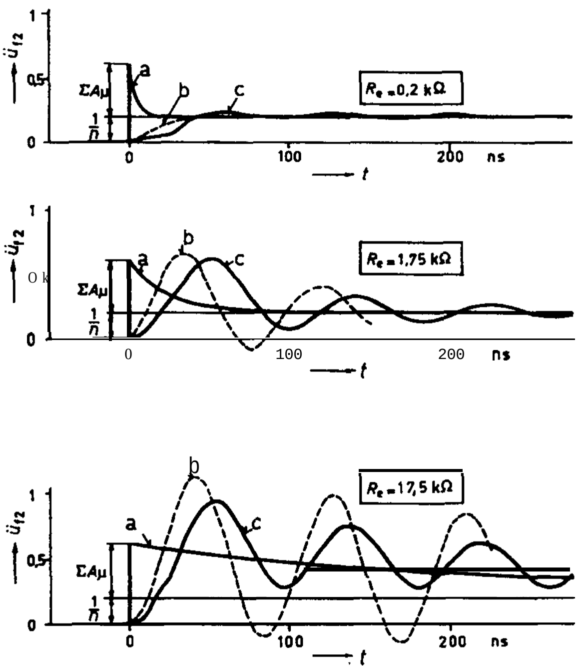


Fig. 8

Behaviour on square wave impulse of: (a) equivalent diagram of Fig.7, overvoltages (*b*) calculated according to equation (11) and overvoltages (c) recorded on the oscilloscope from the second CSG.

Parameter: ohmic value of discharging resistors R_e

$K = C = 5$ pF; $K_{vibr.} = 15$ pF; $L = 5\,\mu$H; $(n+1) = 6$

$\frac{1}{n} + \Sigma A_\mu = \ddot{u}_{f2}\ (t = 0)$ according to equation (5) results, the first state of the overvoltage is described with satisfactory precision by a solution approached for small values of t. This solution is obtained by the propriety of the Laplace equation according to which the behaviour in the beginning range for $t \to 0$ corresponds to the behaviour of the image function for $p \to \infty$. A simple transformation is enough in order to know the behaviour of function Z (p) for $p \to \infty$

Factor $\frac{C}{p R_e}$ is negligible with regard to $\left(CK + \frac{C^2}{4}\right)$ if p is large enough. We thus have

$$Z(p) = \frac{1}{p\sqrt{CK + \frac{C^2}{4}}} \qquad \text{für } p \to \infty \tag{8a}$$

For the transitory value g(p), the limit value to be considered is

$$\cosh g = 1 + \frac{1}{2\left(\frac{K}{C} + \frac{1}{p R_e C}\right)}$$

$$\cosh g \approx 1 + \frac{C}{2K} \qquad \text{für } p \to \infty$$

thus

$$\underset{p \to \infty}{g} \approx \operatorname{arcosh}\left(1 + \frac{C}{2K}\right) \tag{3a}$$

Equations (3a) and (8a) simplify equation (7) to a point where we obtain a clear solution for small values of t for voltage $u_{b1}(t)$

$$\frac{U_0(p)}{pL} - u_{b1}\left(\frac{1}{pL} + \frac{1}{R_e} + p K_1\right) = u_{b1}\, p\, K_2 \tag{7a}$$

with

$$K_2 = \sqrt{CK + \frac{C^2}{4}}\ \operatorname{ctgh}\left[n \cdot \operatorname{arcosh}\left(1 + \frac{C}{2K}\right)\right]$$

Equation (7a) enables us to easily determine the original function $u_{b1}(t)$. According to whether expression $\frac{4 R_e^2 K_3}{L}$ is greater, smaller, or equal to 1, the voltage is oscillatary or aperiodically damped. We are primarily interested in the oscillatory solution. If we define

$$U_0(p) = 1$$

we obtain

$$u_{b1}(t) = 1 - e^{\alpha t}\left(\frac{\sin \beta t}{\sqrt{\frac{4 R_e^2 K_3}{L} - 1}} + \cos \beta t\right) \tag{9}$$

with

$$\alpha = -\frac{1}{2 R_e K_3}; \quad \beta = \sqrt{\frac{1}{L K_3} - \frac{1}{4 R_e^2 K_3^2}}; \quad K_3 = K_1 + K_2$$

The passage from equation (7) to the approached equation (7a) and its resolution into equation (9) can be physically explained as follows:

Only the capacitances C and K of the iterative network, which represents the generator part which has not yet fired, are active immediately after the firing of the first CSG.

$$K_2 = \sqrt{CK + \frac{C^2}{4}}\ \operatorname{ctgh}\left[n \cdot \operatorname{arcosh}\left(1 + \frac{C}{2K}\right)\right]$$

is the input capacitance of the chain with n links, short-circuited at the end.

The first term $\sqrt{CK + \frac{C^2}{4}}$ is the input capacitance of the corresponding chain with infinite length. For small values of t, the reaction of the unfired portion of the generator on the first stage is due only to the increase of stray capacitance K_1 by the value of input capacitance K_2 of said portion.

The sum of stray capacitances acting upon the first stage constitutes, with its self-inductance L, an oscillating circuit excited after the firing of the first CSG. The oscillations are damped by resistor R_e connected in parallel with this circuit. The lower the ohmic value of the discharging resistor, the higher the damping of the oscillation; and this fact explains why the oscillation conditions are different for different values of R_e in Fig. 8.

The variation of the overvoltages on the CSG can be easily calculated by means of the integral equation of Duhamel which gives the relation between the reaction of a linear circuit to any impulse voltage and the behaviour of the transmission of a rectangular wave.

This is described by equation (5) and, in the case which interests us presently, any impulse voltage u_{b1} (t) according to equation (9). We thus have:

$$u_{fk} = u_{b1}(t) \cdot u_{fk}^{r}(+0) + \int_0^t u_{b1}(\tau) \cdot \frac{d u_{fk}^{r}(t-\tau)}{d(t-\tau)}\, d\tau \tag{10}$$

$$k = 1, 2, 3, \ldots n, (n+1)$$

With equations (5) and (9), we finally obtain the equation:

$$u_{fk} = \frac{1}{n} + e^{\alpha t}(B_1 \sin \beta t + B_2 \cos \beta t) + \sum_{\mu=1}^{n-1} B_3 \cdot e^{\frac{-t}{T_\mu}} \tag{11}$$

The values α, β and T_μ are the same as for the calculation of the impulse square wave transmission behaviour according to equation (5). Constants B_1, B_2 and B_3 depend upon the number of generator stages ($n + 1$), stray capacitances C and K, as well as discharging resistors R_e. The dependance of constants B_1, B_2, B_3 on their parameters being rather complicated, the numerical interpretation of equation (11) is better effected with a computer.

If we put $k = 2$, we obtain the overvoltage at the second CSG of the short-circuited generator. A comparison of the oscillograms in Fig. 8 with the calculated voltages according to equation (11) shows satisfactory agreement between calculation and measurement, taking into consideration the first approximation assumed for establishing the equivalent circuit.

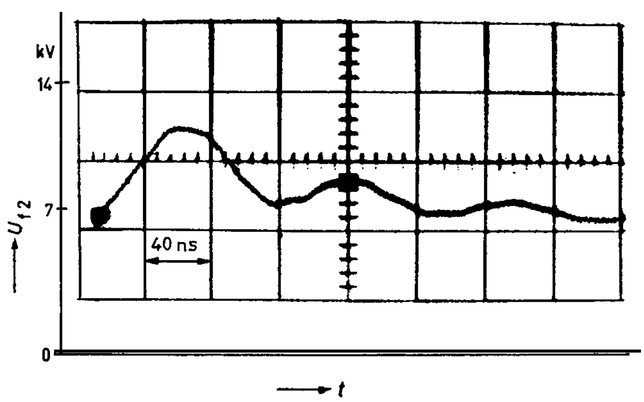


Fig. 9
Voltage u_{f2} on the second CSG after the firing of the first CSG at a charging voltage of 7 kV.
The second CSG does not fire.

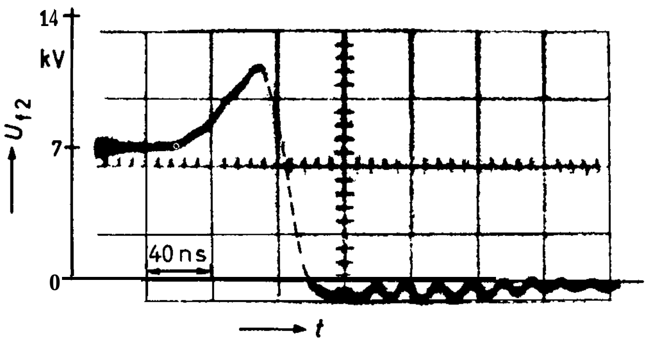


Fig. 10
Voltage u_{f2} on the second CSG after the firing of the first CSG at a charging voltage of 7 kV.
The second CSG fires near the crest of the overvoltage.

8. Firing of the Second CSG

We must still verify whether the transient phenomena, after the firing of the first CSG, really provoke the firing of the other CSG. The testing equipment for a charging voltage of a few kilovolts differs very little from the equipment used with low-voltage tests. The impulse generator is placed inside a copper cylinder which limits the electromagnetic field space due to its firing and avoids interference in the sensitive oscilloscope.

The voltage dividers are short pieces of coaxial cable doubly screened. The capacitance between conductor and first screen acts as primary capacitor and the capacitance between the two screens as secondary capacitor of a voltage divider. These dividers are brought to the generator through openings in the copper cylinder. Fig. 9 shows the oscillogram of an overvoltage u_{f2} of a six-stage generator operating at no-load. taken with the above arrangement. While recording this oscillogram, the distance of the second CSG was adjusted in such a way that it did not fire, even under the influence of the overvoltage. The modification noted, compared with u_{f2} in Fig. 3, results from the earth capacitance increase caused by the voltage dividers and the cylinder acting as screen. The overvoltage characteristic is. nevertheless, the same as in the case of the low-voltage tests. If the gap length of the second CSG is reduced, it will finally fire under the influence of the overvoltage. Fig. 10 shows the oscillogram of the voltage variation on CSG f2 which fires. The voltage collapses to zero from the peak of the overvoltage. During a second firing, a time-interval dispersion occurs between the beginning of the overvoltage and the voltage collapse. This dispersion is influenced by the spark intensity at the first CSG.

9. Summary

The transient phenomena evidenced after the firing of the first CSG in the Marx multiplying circuit have been examined experimentally. The results of these experiments can be represented in the form of a simple equivalent diagram. The results are very similar to the voltage conditions in a resistive divider for the measurement of steep impulse waves. The equivalent diagram consists of a resistor chain with longitudinal and transversal capacitances. This chain is constituted by the interstage resistors in the generator. The discharging resistors play the most important part. The longitudinal and transversal capacitances in the chain are formed by the interstage stray capacitances as well as the capacitances between stages and earth.

After the firing of the first CSG, this iterative network is supplied from the impulse capacitor of the first stage of the generator. The mostly capacitive generator load acts, regarding the considered swift transient phenomena, as a short circuit at the chain output. The longitudinal voltages at the various chain links appear as overvoltages at the corresponding CSG. The firing of a CSG by such an overvoltage is recorded by an oscilloscope

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